MAT1320-LINEAR ALGEBRA FINAL EXAM QUESTIONS		A
Name Surname: ANSWER KEY		**Group No:**
Student No:		**Duration:** 90 mins.
Department:		**Date:** Monday, Jan 17, 2022
Lecturer:		**Signature:**

! Attention: *The 9th article of Student Disciplinary Regulations of Higher Education Council (YÖK) Law No. 2547 states that people who are* ***"cheating or helping to cheat or attempt to cheat in exams"*** *will be punished by suspension of one or two semesters.*

1. Let $V=\mathbb{R}^2$ and $A=\{(1,0),(0,1)\}$, $B=\{(2,0),(1,3)\}$, $C=\{(1,-3),(2,4)\}$ be subsets of V. The vector $v=(8,6)\in V$ can be written as the linear combination of ....

   a) Only A b) A and B c) Only C
   d) B and C e) A, B and C

2. Which of the following statement(s) are correct for the homogeneous system

$$\begin{aligned} 2x_1+3x_2+7x_3&=0\\ -2x_1-4x_3&=0\\ x_1+2x_2+4x_3&=0 \end{aligned}$$

   I. The system can be solved by using Cramer's rule.
   II. The system has infinitely many solutions.
   III. The system has only trivial (zero) solution.

   a) I and II b) Only II c) Only III
   d) I and III e) Only I

3. Let $A=\begin{bmatrix} 0 & a+b & c+2\\ a & 2 & c\\ 4 & a+b & 4\end{bmatrix}$ be a symmetric matrix. Which of the following statement(s) are correct for the matrix $B=\begin{bmatrix} b & a & -2\\ b-a & 0 & 1\\ c & -1 & b\end{bmatrix}$?

   I. B is a skew-symmetric matrix.
   II. B^2 is a symmetric matrix.
   III. $\mathrm{Tr}(B)=\mathrm{Tr}(A)$.

   a) Only I b) I and II c) II and III
   d) I and III e) All of them

4. What is the value of the determinant $\begin{vmatrix} 1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix}$?

   a) $(a+b)(a+c)(b+c)$ b) $(a-b)(c-a)(c-b)$
   c) $(b-a)(a-c)(c-b)$ d) $(b-a)(c-a)(c-b)$
   e) $(b-a)(c-a)(b-c)$

5. Let $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid y = x + z, \text{ where } x, y, z \in \mathbb{R} \right\}$ be the subspace of $\mathbb{R}^3$. What is the dimension of S ?

a) 1 b) 2 c) 3 d) 4

e) None of them

6. Which of the following subsets are subspaces of the given vector spaces ?

$$\mathcal{Y} = \left\{ \begin{bmatrix} x \\ x^2 \end{bmatrix} \mid x \in \mathbb{R} \right\} \subset \mathbb{R}^2$$

$$\mathcal{T} = \left\{ \begin{bmatrix} x \\ x+1 \\ 0 \end{bmatrix} \mid x \in \mathbb{R} \right\} \subset \mathbb{R}^3$$

$$\mathcal{U} = \left\{ \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} \mid x \in \mathbb{R} \right\} \subset \mathbb{R}^3$$

a) Only $\mathcal{Y}$ b) Only $\mathcal{T}$ c) Only $\mathcal{U}$

d) $\mathcal{Y}$ and $\mathcal{T}$ e) $\mathcal{T}$ and $\mathcal{U}$

7. Let $P(-1,3,2), Q(2,1,3)$ and $R(5,-2,4)$ be points in $\mathbb{R}^3$. Assume that $\overrightarrow{\mathbf{u}} = \overrightarrow{PQ}$ and $\overrightarrow{\mathbf{v}} = \overrightarrow{PR}$. Which of the following vector is perpendicular (orthogonal) to both $\overrightarrow{\mathbf{u}}$ and $\overrightarrow{\mathbf{v}}$?

a) $\vec{i} + \vec{j} + \vec{k}$ b) $\vec{i} + 2\vec{j} - 3\vec{k}$ c) $\vec{j} + \vec{k}$

d) $\vec{i} - 3\vec{k}$ e) $3\vec{i} - \vec{j} + 2\vec{k}$

8. Let $M_{n \times n}$ denote the vector space of all $n \times n$ real matrices. Consider the subset

$$W = \left\{ \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} \in M_{2\times 2} \mid a + b + c = 0 \text{ where } a, b, c \in \mathbb{R} \right\}$$

Which of the following statements are always true?

I. The set W is a subspace of $M_{2\times 2}$.

II. $B = \left\{ \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix} \right\}$ forms a basis for W.

III. $\dim(W) = 2$.

a) Only I b) Only II c) Only III

d) I and II e) I and III

9. Let $\vec{a}, \vec{b}, \vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^n$ and suppose the following scalar (dot/inner) products hold:

$$\vec{a}\cdot\vec{a}=2,\quad \vec{a}\cdot\vec{b}=1,\quad \vec{a}\cdot\vec{v}=-1,\quad \vec{a}\cdot\vec{w}=-3$$
$$\vec{b}\cdot\vec{b}=6,\quad \vec{b}\cdot\vec{v}=4,\quad \vec{b}\cdot\vec{w}=-4,\quad \vec{v}\cdot\vec{v}=7$$
$$\vec{v}\cdot\vec{w}=2,\quad \vec{w}\cdot\vec{w}=2$$

Which of the following is equal to $(2\vec{a}-\vec{v})\cdot(\vec{w}+\vec{b})$?

a) -10 b) -14 c) -2 d) 2 e) 14

10. Let $A = \begin{bmatrix} -2 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 2 \end{bmatrix}$. Which of the following is the $(3,3)$-entry of the inverse matrix A^{-1} ?

a) 0 b) -2 c) 1 d) -1 e) 2

11. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $ad-bc=5$ and $a+d=6$, which of the following is the characteristic polynomial of A ?

a) $p(\lambda)=\lambda^2-6\lambda+5$ b) $p(\lambda)=3\lambda^2-4\lambda+6$
c) $p(\lambda)=\lambda^2-5\lambda+6$ d) $p(\lambda)=2\lambda^2-3\lambda+6$
e) $p(\lambda)=\lambda^2+5\lambda-6$

12. Let B be an invertible matrix with an appropriate size and $A = \begin{bmatrix} -1 & 2 \\ 3 & 3 \end{bmatrix}$. If the equation $A^{-1}B^2 = A^3B$ holds, what is B ? (Hint: Cayley-Hamilton theorem can be used.)

a) $16A+24I_2$ b) $32A+34I_2$ c) $44A+117I_2$
d) $76A+184I_2$ e) $96A+196I_2$

13. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$. Which of the following can be the eigenvector associated with the largest eigenvalue of the matrix A ?

a) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ b) $\begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$ c) $\begin{bmatrix} 15 \\ 6 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$ e) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

14. Let $S = \{(1,0,1),(1,1,0),(0,0,1)\}$ and $T=\{w_1,w_2,w_3\}$ be ordered bases for $\mathbb{R}^3$. Suppose that the transition matrix from T to S is $[M]_T^S = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$.

Which of the following is T ?

a) $\{(3,2,0),(2,1,0),(3,1,2)\}$
b) $\{(1,0,1),(2,1,3),(3,0,1)\}$
c) $\{(1,1,1),(1,1,3),(3,3,1)\}$
d) $\{(1,2,1),(1,1,2),(2,2,1)\}$
e) $\{(2,0,2),(1,3,0),(3,0,1)\}$

15. For what value(s) of t, the set $\{(1,0,2),(0,t,1),(t^2,0,2)\}$ forms a basis for $\mathbb{R}^3$?

a) $t \in \mathbb{R}-\{0,1\}$
b) $t \in \mathbb{R}-\{0,-1\}$
c) $t=-1$
d) $t \in \{-1,0,1\}$
e) $t \in \mathbb{R}-\{-1,0,1\}$

16. Which of the following matrices is the transition matrix $[M]_S^T$ from basis S to basis T of $\mathbb{R}^2$ where

$$S=\{(-3,2),(4,-2)\},\ T=\{(-1,2),(2,-2)\}\ ?$$

a) $\begin{bmatrix}-1 & 2\\ -2 & 3\end{bmatrix}$
b) $\begin{bmatrix}-1 & -2\\ -2 & 3\end{bmatrix}$
c) $\begin{bmatrix}-1 & 2\\ -2 & -3\end{bmatrix}$
d) $\begin{bmatrix}1 & 2\\ -2 & -3\end{bmatrix}$
e) $\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$

17. Let P_2 be the vector space of all polynomials of degree ≤ 2 and a zero polynomial. Which of the following subsets of P_2 are subspaces ?

$$\mathcal{M}=\{at^2+bt+c : b=c=0\}$$
$$\mathcal{A}=\{at^2+bt+c : b=2c\}$$
$$\mathcal{T}=\{at^2+bt+c : a+b+c=2\}$$

a) Only $\mathcal{T}$
b) $\mathcal{M}$ and $\mathcal{A}$
c) $\mathcal{M}$ and $\mathcal{T}$
d) $\mathcal{A}$ and $\mathcal{T}$
e) All

18. Which of the following is the vector $(\overrightarrow{u}\times\overrightarrow{w})\times\overrightarrow{v}$, where $\overrightarrow{u}=(1,0,0)$, $\overrightarrow{v}=(0,1,0)$, $\overrightarrow{w}=(0,0,1)$?

a) $(0,-1,0)$
b) $(0,1,0)$
c) $(0,0,1)$
d) $(0,0,-1)$
e) $(0,0,0)$

19. If $\lambda = 1$ is one of the eigenvalues of the matrix $A = \begin{bmatrix}3 & a\\ b & -5\end{bmatrix}$, which of the following might be another eigenvalue for A ?

a) 2
b) 3
c) -1
d) -3
e) -2

20. Which of the following matrices are in reduced row echelon form?

$$\mathcal{A}=\begin{bmatrix}1&2&0&0&1\\0&0&1&2&3\\0&0&0&0&0\end{bmatrix},\ \mathcal{B}=\begin{bmatrix}1&2&0&4\\0&0&0&0\\0&0&-1&3\end{bmatrix},$$

$$\mathcal{C}=\begin{bmatrix}1&0&3&4\\0&1&-2&5\\0&1&2&2\\0&0&0&0\end{bmatrix},\ \mathcal{D}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

a) $\mathcal{B}$ and $\mathcal{C}$
b) Only $\mathcal{D}$
c) $\mathcal{A}$ and $\mathcal{D}$
d) Only $\mathcal{A}$
e) Only $\mathcal{B}$